

Name:

Answers/solutions

Directions: Try each problem without your calculator, then use your calculator ONLY IF you feel you have truly tried everything possible without your calculator.

1. Determine an equation of a line containing the points  $(3, -5)$  and  $(3, 1)$ .

$$m = \frac{1 - (-5)}{3 - 3} = \frac{6}{0} = \text{undefined} \quad \text{vertical line}$$

$$x = 3 \quad \checkmark$$

2. Line A has equation  $y + 10 = \frac{-3}{5}(x - 11)$ . Line B contains the point  $(-10, 14)$  and is perpendicular to line A. Determine an equation for line B in general form (standard form).

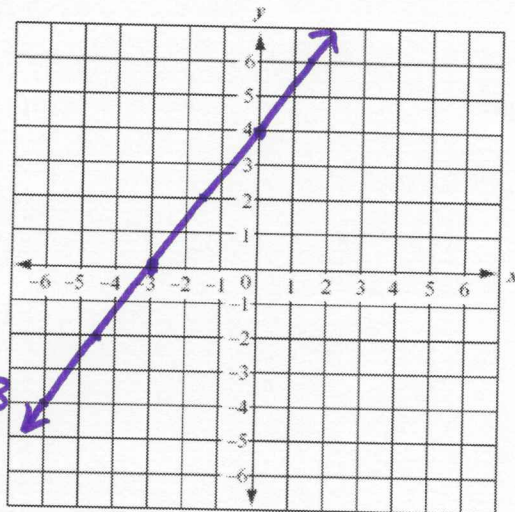
$$\begin{aligned} \text{slope of line B is } m &= \frac{5}{3} \\ y - 14 &= \frac{5}{3}(x + 10) \\ y - 14 &= \frac{5}{3}x + \frac{50}{3} \end{aligned}$$

$$\begin{aligned} 3y - 42 &= 5x + 50 \\ -5x + 3y &= 92 \end{aligned}$$

3. Graph  $4x - 3y = -12$  (plot several points and draw your line neatly)

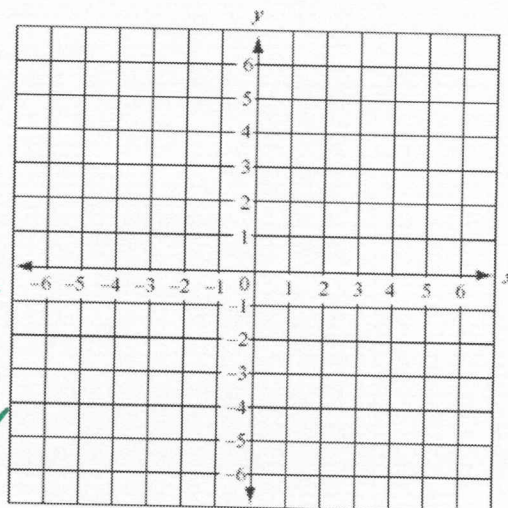
$$\begin{aligned} \text{If } x = 0, \text{ we have } (0, y) \\ 4(0) - 3y &= -12 \quad -3y = -12 \quad y = 4 \\ (0, 4) \end{aligned}$$

$$\begin{aligned} \text{If } y = 0, \text{ we have } (x, 0) \\ 4x - 3(0) &= -12 \quad 4x = -12 \quad x = -3 \\ (-3, 0) \end{aligned}$$

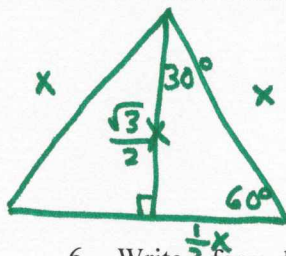


4. A line contains the points plotted below. Write an equation of the line in point-slope form, slope-intercept form, and general form (standard form).

I do not recall what line I drew



5. Write a formula for the height of an equilateral triangle as a function of its side length,  $x$ .



$$h(x) = \frac{\sqrt{3}}{2}x$$

6. Write a formula for the volume of a sphere as a function of the sphere's radius,  $r$ .

$$V(r) = \frac{4}{3}\pi r^3$$

7. Identify the domain and range of the function  $f(x) = 9 - x^2$ .

$$D: (-\infty, \infty)$$

$$R: (-\infty, 9]$$

8. Identify the domain and range of the function  $g(x) = \sqrt{5-x}$ .

$$\begin{aligned} 5-x &\geq 0 \\ -x &\geq -5 \\ x &\leq 5 \end{aligned}$$

$$D: (-\infty, 5]$$

$$R: [0, \infty)$$

9. Identify the domain and range of the function  $k(x) = \sqrt{16-x^2}$ .

$$D: [-4, 4]$$

$$R: [0, 4]$$

$$16-x^2 \geq 0$$

$$x \geq -4 \text{ and } x \leq 4$$

$$-4 \leq x \leq 4$$

10. Identify the domain and range of the function  $p(x) = \sqrt{-x}$ .

$$D: (-\infty, 0]$$

$$R: [0, \infty)$$



11. Identify the domain and range of the function  $h(x) = \sqrt[3]{x}$ .

$$D: (-\infty, \infty)$$

$$R: (-\infty, \infty)$$

12. Determine whether the function  $f(x) = x^{\frac{1}{3}}$  is even, odd, or neither.

$$f(x) = -f(-x) \quad \text{or} \quad -f(x) = f(-x)$$

$$\text{example: } f(27) = 3 \quad f(-27) = -3 \quad (27, 3) \text{ \& } (-27, -3)$$

odd function

If  $(x, y)$  is on the graph, then so is  $(-x, -y)$

13. Determine whether the function  $g(x) = x^{\frac{2}{3}}$  is even, odd, or neither.

$$f(8) = 4 \quad f(-8) = 4 \quad f(x) = f(-x)$$

If  $(x, y)$  is on the graph, then so is  $(-x, y)$

$\rightarrow (8, 4) \text{ \& } (-8, 4)$  are on the graph

even function

14. Determine whether the function  $k(x) = x^4 - 3x^2$  is even, odd, or neither.

$$K(x) = K(-x)$$

$$\text{example: } K(2) = 4 \text{ \& } K(-2) = 4$$

$(2, 4) \text{ \& } (-2, 4)$  are on the graph

If  $(x, y)$  is on the graph, so is  $(-x, y)$

Even Function

15. Determine whether the function  $p(x) = \frac{1}{x}$  is even, odd, or neither.

$$p(x) = -p(-x) \quad \text{or} \quad -p(x) = p(-x)$$

$$\text{example: } p(2) = \frac{1}{2} \text{ \& } p(-2) = -\frac{1}{2} \quad (2, \frac{1}{2}) \text{ \& } (-2, -\frac{1}{2})$$

If  $(x, y)$  is on the graph, then so is  $(-x, -y)$

Odd Function

16. Determine whether the function  $m(x) = x^3 - 3x$  is even, odd, or neither.

$$m(x) = -m(-x) \quad \text{or} \quad -m(x) = m(-x)$$

$$\text{example: } m(3) = 18 \text{ \& } m(-3) = -18$$

$(3, 18) \text{ \& } (-3, -18)$  are both on the graph

If  $(x, y)$  is on the graph, then so is  $(-x, -y)$

17. Given the piecewise function  $f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 2 \\ 2x - 1 & \text{if } x > 2 \end{cases}$ , determine the following:

a. the value of  $f(2) = (2)^2 - 1 = 3$

b. if the function is continuous at  $x = 2$

$(2)^2 - 1 = 3$     $2(2) - 1 = 3$    yes, because both parts of the function equal 3 at the split at  $x = 2$

c. the domain of the function

$x \leq 2$  or  $x > 2$  is the same as  $(-\infty, \infty)$   
 $D: (-\infty, \infty)$

d. the range of the function

$R: [-1, \infty)$

18. Without a calculator, draw an approximate graph of each function:

a.  $f(x) = e^x$

see attached page

b.  $f(x) = e^{-x}$

reflection of  $f(x) = e^x$  in the y-axis  
see attached page

c.  $f(x) = e^x - 3$

translation (shift) 3 units down  
see attached page

d.  $f(x) = -e^x$

reflection of  $f(x) = e^x$  in the x-axis  
see attached page

19. Use a graph on your calculator to find the zeros of the function  $f(x) = 3^x - 7$

Method 1:  $y_1 = 3^x - 7$  graph use "zero" in "calc" menu

Method 2:  $y = 3^x - 7$     $0 = 3^x - 7$     $3^x = 7$     $\ln 3^x = \ln 7$   
 $x \ln 3 = \ln 7$     $x = \frac{\ln 7}{\ln 3}$     $x \approx 1.771$



20. Determine the domain and range of the exponential function  $f(x) = 2e^x + 3$

$$D: (-\infty, \infty)$$

$$R: (3, \infty)$$

21. The population of Silver Run in the year 1890 was 6250. Assume the population increased at a rate of 2.75% per year.

- a. Estimate the population in 1915.

$$A(t) = A_0 (1 + 0.0275)^t \text{ where } t = \text{years since 1890}$$

$$A(25) = 6250(1.0275)^{25} \approx 12,314.76 \approx 12,315 \text{ People}$$

- b. Approximate the year when the population reached 50,000.

$$50000 = 6250(1.0275)^t$$

$$8 = 1.0275^t$$

$$\ln 8 = \ln 1.0275^t$$

$$\ln 8 = t \ln 1.0275$$

$$t = \frac{\ln 8}{\ln 1.0275}$$

$$t \approx 76.65$$

$$1890 + 77 = 1967$$

approximately

22. Determine how much time is required for an investment to double in value if interest is earned at the rate of 6.25% compounded annually.

$$A(t) = A_0(1 + r)^t \Rightarrow A(t) = A_0(1.0625)^t$$

$$2A_0 = A_0(1.0625)^t \Rightarrow 2 = (1.0625)^t$$

$$\ln 2 = \ln(1.0625)^t \Rightarrow \ln 2 = t \ln(1.0625) \Rightarrow t = \frac{\ln 2}{\ln 1.0625}$$

23. The number of bacteria in a petri dish culture after  $t$  hours is  $B(t) = 100e^{0.693t}$ .

$$t \approx 11.433 \text{ years}$$

- a. What was the initial number of bacterial present?

$$B(0) = 100e^{0.693(0)}$$

$$B(0) = 100e^0$$

$$B(0) = 100$$

- b. How many bacteria are present after 6 hours?

$$B(6) = 100e^{0.693(6)}$$

$$B(6) \approx 6,394 \text{ bacteria}$$

- c. Approximately when will the number of bacteria be 200? In other words, estimate the doubling time of the bacteria.

$$200 = 100e^{0.693t}$$

$$2 = e^{0.693t}$$

$$\ln 2 = \ln(e^{0.693t})$$

$$\ln 2 = 0.693t$$

$$\frac{\ln 2}{0.693} = t$$

$$t \approx 1 \text{ hour}$$